

# Theory Analysis on Magnetoelectric Voltage Coefficients of the Terfenol-D/PZT Composite Transducer

Shuxiang Dong, JieFang Li, and Dwight Viehland  
Materials Science & Engineering, Virginia Tech, Blacksburg, VA24061

## Abstract

The magnetoelectric response of magnetostrictive/piezoelectric laminate composites has been investigated. Based on the piezoelectric and piezomagnetic constitutive equations, and motion equation for the composite plate, the magnetoelectric equivalent circuit has been derived, and the magnetoelectric coefficients predicted. Prototype devices of, as an application example, Terfenol-D/PZT composite plates were fabricated, and experimental values for the magnetoelectric voltage coefficient coincided with the predicted ones.

## I. Introduction

The magneto-electric (ME) effect is a polarization response to an applied magnetic field  $H$ , or conversely a magnetization response to an applied electric field  $E$ . [1] Magneto-electric materials have been studied [2-7] such as  $\text{BiFeO}_3$ ,  $\text{Pb}(\text{Fe}_{1/2}\text{Nb}_{1/2})\text{O}_3$ , and  $\text{Cr}_2\text{O}_3$ ; however to date, a single phase material with a high inherent coupling between magnetization  $M$  and polarization  $P$  has yet to be found.

Magneto-electric behavior has also been studied as a composite effect in multi-phase systems consisting of both piezoelectric and magnetostrictive materials [8-21]. Piezoelectric/ magnetostrictive composites have been the topic of numerous investigations, both experimentally and analytically. Various composite connectivities of two phases have been studied, including 3-3 (i.e., ceramic-ceramic particle) and 2-2 (laminate). These studies have confirmed the existence of ME effects in composites; however, the magnitude of the coupling is low.

Investigations of piezoelectric-magnetostrictive laminate composites have previously been reported using constitutive equations [10, 20,21,22]. The ME coefficient of 2-2 type laminates has been approximated for the transverse magnetized/ transverse polarized (T-T) mode of operation [10,20]. Unfortunately, the approach is suitable only for dc magnetic biases; furthermore, the approach does not lend itself to the design of laminates operating in other modes. This constitutive approach combined both piezoelectric and piezomagnetic constitutive equations. But, it did not consider the coupled equation of motion for the laminate, and consequently ignored the magnetic energy transduced into electrical energy by this coupled motion. Any quantitative prediction of ME coefficients and the design of other ME laminate configurations must be based on these considerations.

An equivalent circuit approach could be used to model the ME effect. Even though such methods are well known for both electromechanical and magnetomechanical couplings [23,24], they have yet to be applied to ME coupling. In this paper, an equivalent circuit method is used to analyze the ME behavior of laminates of magnetostrictive terfenol-D ( $\text{Tb}_x\text{Dy}_{1-x}\text{Fe}_2$ ) and piezoelectric  $\text{Pb}(\text{Zr}_{1-x}\text{Ti}_x)\text{O}_3$  (PZT). The approach is based on piezoelectric and magnetostrictive constitutive equations, where the layers are strain-stress coupled. An equation of motion due to a magnetic excitation is developed, from which the transduced electrical energy is obtained.

## II. Working Modes of Magneto-Electric Laminate Composites

Terfenol-D has a large magnetostrictive strain, which is anisotropic and a function of  $\vec{H}$ . Thus, it has large magnetostrictive effects only for specific modes. Previous investigations of magnetostrictive/piezoelectric laminates have used terfenol-D in the transverse mode (T-mode), where  $\vec{H}$  is applied perpendicular to the magnetization  $\vec{M}$  direction. One problem associated with the T-mode is that the magnetostrictive constant  $d_{31,m}$  is small. We find that terfenol-D layers (grain-oriented in the thickness direction) operated in the L-mode ( $\vec{H}$  is parallel to  $\vec{M}$ ) have 8x the strain of that in the T-mode. Consequently, ME effects in laminates which use the L-mode will be significantly larger than those that use the T-mode.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>00 JUN 2003</b>		2. REPORT TYPE <b>N/A</b>		3. DATES COVERED <b>-</b>	
4. TITLE AND SUBTITLE <b>Theory Analysis on Magnetoelectric Voltage Coefficients of the Terfoneol-D/PZT Composite Transducer</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Materials Science &amp; Engineering, Virginia Tech, Blacksburg, VA24061</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release, distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>See also ADM001697, ARO-44924.1-EG-CF, International Conference on Intelligent Materials (5th) (Smart Systems &amp; Nanotechnology)., The original document contains color images.</b>					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>UU</b>	18. NUMBER OF PAGES <b>8</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

Figure 1(a) shows our new laminate design. It consists of a PZT layer poled in the thickness direction, which is sandwiched between two terfenol-D layers magnetized in the length direction. This ME laminate is operated in the L-T mode. The design is different than conventional T-T ones [8-20], which is illustrated in Figure 2(b).

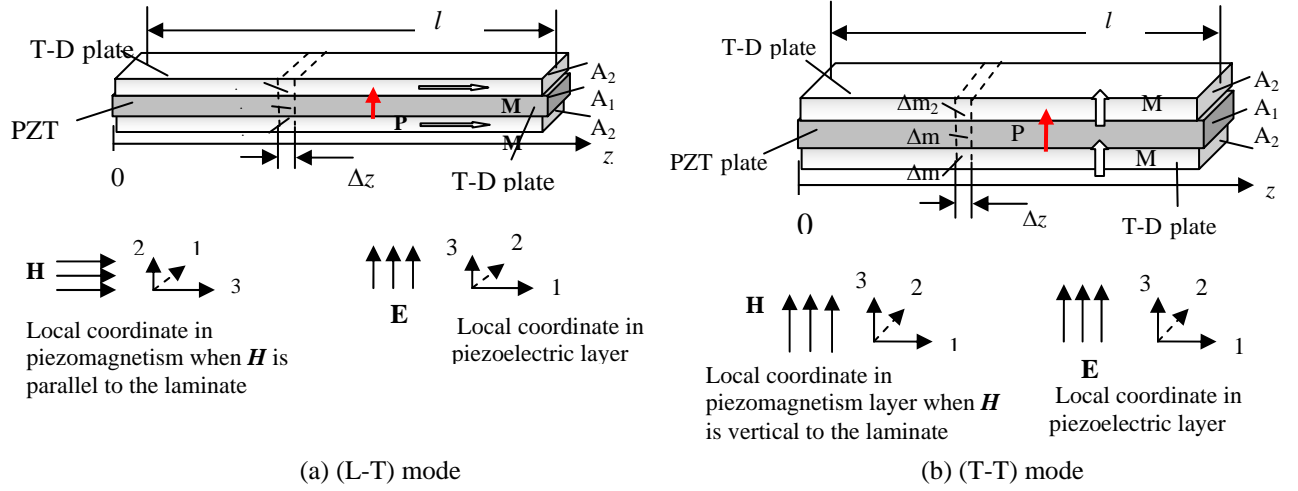


Figure 1. Laminate composite designs. (a) L-T laminate, and (b) T-T laminate.

The magnetostrictive and piezoelectric layers are mutually coupled through strain  $S(z)$  and stress  $T(z)$ . Thus, application of  $\vec{H}$  along the length direction of the magnetostrictive layer (see Figure 1a) puts the piezoelectric one into forced oscillation, generating a voltage across its thickness. [Note, both longitudinal  $d_{33,m}$  and transverse  $d_{31,m}$  magnetostrictive vibration modes will be excited; however, because the laminate is long and thin, the transverse (31) vibration mode can be neglected.] This is the L-T magneto-electric effect. Similarly, application of  $\vec{H}$  along the thickness direction of the magnetostrictive layers (Figure 1b) generates a volt across the thickness of the piezoelectric layer. [Note, when  $\vec{H}$  is applied along this direction in Figure 1(b), the thickness (33) vibration mode can be neglected.] This is the T-T magneto-electric effect, which has previously been investigated [8-20].

### III. Analysis of Magneto-(Elasto)-Electric Coupling

The ME behavior of laminate composites is a product property. Product properties do not naturally occur in single phase materials, but rather are due to the integration of multiple functionalities within a composite. Two sets of linear constitutive equations (for small signal excitation) are required to describe the ME product property in laminate composites, which are mutually coupled by strain  $S(z)$  and stress  $T(z)$ .

#### A. Constitutive Equations

We suppose that the piezoelectric layer is polarized along its thickness direction, and that stress is imposed on this layer by the magnetostrictive (terfenol-D) layers along the length direction, regardless of whether  $\vec{H}$  is applied parallel or perpendicular to the length of the plate. Accordingly, the piezoelectric constitutive equations are

$$S_{1p} = s_{11}^E T_{1p} + d_{31,p} E_3 \quad D_3 = d_{31,p} T_{1p} + \epsilon_{33}^T E_3; \quad (1)$$

where  $D_3$  is the electric displacement,  $\epsilon_{33}^T$  is the permittivity under constant stress,  $s_{11}^E$  is the elastic compliance of the piezoelectric material under constant E,  $d_{31,p}$  is the transverse piezoelectric constant, and  $T_{1p}$  and  $S_{1p}$  are the stress and strain in the length direction of the piezoelectric layer imposed by the magnetostrictive layers. [Note, using the local coordinate system in piezoelectric plate in Figure 1, the local coordinate axis “3” is assumed to be defined by the polarization direction, which is along the thickness. Accordingly, the local coordinate axis “1” is the length direction of the laminate.]

When  $\vec{H}$  is applied parallel to the longitudinal axis of the laminate, a longitudinal (33) strain is excited. [Note, using the local coordinate system in piezomagnetism in Figure 1, the local coordinate axis “3” is assumed to be defined by the magnetization direction, which is along the longitudinal axis of the laminate. Accordingly, the local coordinate axis “1” is the thickness direction of the laminate.]

The piezomagnetic constitutive equations for the (33) longitudinal mode are

$$S_{3m} = s_{33}^H T_{3m} + d_{33,m} H_3 \quad B_3 = d_{33,m} T_{3m} + \mu_{33}^T H_3; \quad (2a)$$

where  $B_3$  is the magnetization along the length direction,  $\mu_{33}^T$  is the permeability under constant stress,  $s_{33}^H$  is the elastic compliance of the magnetostrictive material under constant H,  $d_{33,m}$  is the longitudinal magnetostrictive constant, and  $T_{3m}$  and  $S_{3m}$  are the stress and strain in the longitudinal direction of the magnetostrictive layers imposed by on the piezoelectric layer.

When  $\vec{H}$  is applied parallel to the thickness of the laminate, a transverse (31) strain is excited. The piezomagnetic constitutive equations for the (31) transverse mode are

$$S_{1m} = s_{11}^H T_{1m} + d_{31,m} H_3 \quad B_3 = d_{31,m} T_{1m} + \mu_{33}^T H_3; \quad (2b)$$

where  $d_{31,m}$  is the transverse magnetostrictive constant.

These constitutive equations are linear relationships, which do not account for loss components. Significant nonlinearities in both piezoelectric and magnetostrictive materials are known to exist, in addition to losses. Our approach will treat the individual layers of the laminate using the above given harmonic approximation.

## B. Equation of Motion

Assuming harmonic motion, along a given direction z, it will be supposed that three small mass units ( $\Delta m_1$ ,  $\Delta m_2$  and  $\Delta m_3$ ) in the laminate have the same displacement  $u(z)$  vector given as

$$u_{1p} = u_{im} = u(z), \quad \text{or} \quad S_{1p} = S_{im} = \frac{\partial u}{\partial z}; \quad (3)$$

where  $i=1$  results in the T-T mode ME effect when  $\vec{H}$  is applied in the thickness direction, and where  $i=3$  results in the L-T mode one when  $\vec{H}$  is applied in the length direction. This follows from Figure 2, by assuming that the layers in the laminate act only in a coupled manner. Following Newton's Second Law, we then have

$$(\Delta m_1 + \Delta m_2 + \Delta m_3) \frac{\partial^2 u}{\partial t^2} = \Delta T_{1p} (A_1) + \Delta T_{im} (2A_2) \quad (4)$$

where  $\Delta m_1 = \Delta m_3 = \rho_p A_1 \Delta z$ ,  $\Delta m_2 = \rho_m A_2 \Delta z$ , and  $\rho_p$  and  $\rho_m$  are the mass densities of the piezoelectric and magnetostrictive layers respectively.

For a given laminate width  $w_{lam}$  and thickness  $t_{lam}$ , the total cross-sectional area of the laminate is the sum of the layer areas  $A_{lam} = A_p + A_m = t_{lam} w_{lam}$ , and the total thickness is the sum of the layer thicknesses  $t_{lam} = t_p + 2t_m$ . The equation of motion can then be rewritten as

$$\rho \frac{\partial^2 u}{\partial t^2} = n \frac{\partial T_{i,m}}{\partial z} + (1-n) \frac{\partial T_{1,p}}{\partial z} \quad (0 < n < 1); \quad (5)$$

where  $n = A_2/A = t_m/t_{lam}$  is a geometric factor, and  $\rho = \frac{\rho_p A_1 + \rho_m (2A_2)}{A}$  is the average mass density of the laminate.

Substituting equation 1, the stress T from equation 2 and the definition of strain S into equation 5, the equation of motion can be linked to the mean sound velocity  $\bar{v}$  of the laminate as

$$\frac{\partial^2 u}{\partial t^2} = \bar{v}^2 \frac{\partial^2 u}{\partial z^2} \quad (6)$$

$$\text{where } \bar{v}^2 = \left( \frac{n}{s_{ii}^H} + \frac{1-n}{s_{11}^E} \right) / \rho = \begin{cases} 1/\rho_p s_{11}^E, & n=0 \\ 1/\rho_m s_{ii}^H, & n=1 \end{cases} \quad (ii = 11 \text{ or } 33) \quad (7)$$

For terfenol-D,  $s_{11}^H \gg s_{33}^H$ , thus it is essential to differentiate the longitudinal and transverse cases. When  $\vec{H}$  is applied perpendicular or parallel to z, the transverse (T-T mode) and longitudinal (L-T mode) mean sound velocities  $\bar{v}$  are respectively given as

$$\bar{v}_{T-T}^2 = \left( \frac{n}{s_{11}^H} + \frac{1-n}{s_{11}^E} \right) / \bar{\rho} \quad \text{and} \quad \bar{v}_{L-T}^2 = \left( \frac{n}{s_{33}^H} + \frac{1-n}{s_{11}^E} \right) / \bar{\rho}. \quad (8)$$

Under harmonic oscillation, equation 6 then simplifies to

$$\frac{\partial^2 u}{\partial z^2} + k^2 u = 0, \quad k^2 = \frac{\omega^2}{\bar{v}^2}; \quad (9)$$

where the wave number k equals  $\frac{\omega}{\bar{v}_{L-T}}$  and  $\frac{\omega}{\bar{v}_{T-T}}$  for the L-T and T-T modes respectively, and  $\omega$  is the angular frequency.

### C. Magneto-(elasto)-electric equivalent circuit

By combining piezoelectric and piezomagnetic constitutive equations, solutions to the equation of motion can be derived for the L-T and T-T magneto-electric modes, given as

$$F_1 = Z_1 \dot{u}_1 + Z_2 (\dot{u}_1 - \dot{u}_2) + \varphi_p V + \varphi_m H_3 \quad (10a)$$

$$F_2 = -Z_1 \dot{u}_2 + Z_2 (\dot{u}_1 - \dot{u}_2) + \varphi_p V + \varphi_m H_3 \quad (10b)$$

$$I_p = j\omega C_0 V + \varphi_p (\dot{u}_2 - \dot{u}_1); \quad (10c)$$

where  $F_1$  and  $F_2$ , and  $\dot{u}_1$  and  $\dot{u}_2$  are the forces and the displacement speeds at the two end surfaces of the laminate, respectively;  $V$  and  $I_p$  are induced voltage and current from the piezoelectric layer due to ME coupling, respectively;  $Z_1$  and  $Z_2$  are the mechanical characteristic impedances of the composite;  $C_0$  is static (clamped) capacitance of the piezoelectric plate; and  $\varphi_p$  and  $\varphi_m$  are the elasto-electric (electromechanical) and magnetoelastic coupling factors.

A combined magneto-(elasto)-electric (or ME) equivalent circuit for both L-T and T-T modes can now be developed, as given in Figure 2a. In this diagram, the forces  $F_1$  and  $F_2$  at the boundaries  $z=0$  and  $z=l$  act as a “mechanical voltage”, and  $\dot{u}_1$  and  $\dot{u}_2$  act as a “mechanical current”. Without an external magnetic field  $\mathbf{H}$ , the exerted force  $F_1$  and  $F_2$ , which act as “mechanical voltage” and excite “mechanical current”, can induce an electric voltage and current on the piezoelectric plate via piezoelectric effect with coupling factor  $\varphi_p$ . Without external forces  $F_1$  and  $F_2$ , an applied magnetic field  $\mathbf{H}$ , which also acts as a “mechanical voltage”, can induce a “mechanical current” via the magneto-elastic effect with a coupling factor  $\varphi_m$ . In turn, this results in an electrical voltage  $V$  across the piezoelectric layer, due to electromechanical coupling. A transformer with a turn-ratio of  $\varphi_p$  can be used to represent the electromechanical coupling in the circuit. The current  $I_p$  produced across the piezoelectric layer is then coupled to the “mechanical current” ( $\dot{u}_1$  and  $\dot{u}_2$ ), as given in equation 9.

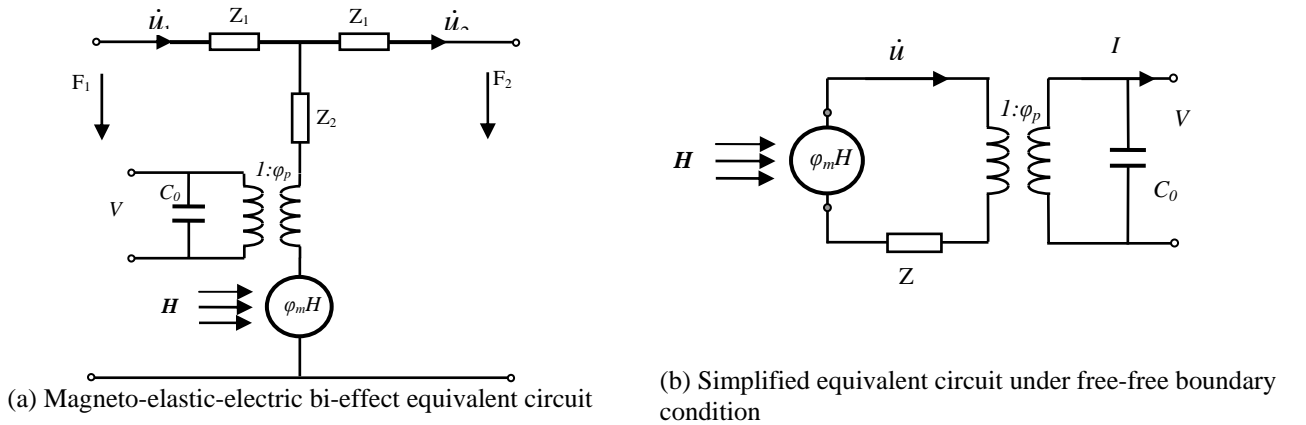


Figure 2. Magneto-elastic-electric bi-effect equivalent circuit for the magnetostrictive/ piezoelectric laminate. In Fig.3b,

$$Z = -\frac{1}{2} j \bar{\rho} \bar{v} A_{lam} c t g \left( \frac{kl}{2} \right).$$

The parameters from the motion/constitutive equations have equivalent electrical analogues, given as

$$Z_1 = j\overline{\rho v} A t g \frac{kl}{2}, Z_2 = \frac{\overline{\rho v} A}{j \sin kl}, \varphi_m = A_2 \frac{d_{3i,m}}{s_{ii}^H}, C_0 = \frac{lw}{t_1} \varepsilon_{33}^S; \varphi_p = \frac{w d_{31,p}}{s_{11}^E}; (i = 1 \text{ or } 3) \quad (11)$$

where  $i=1$  for the T-T mode, and  $i=3$  for the L-T mode. Under free-free boundary conditions,  $F_1=F_2=0$ . In this case, the input and output channels are shorted to ground, and  $Z_2+Z_l/Z_1 = -\frac{1}{2} j\overline{\rho v} A c \tan\left(\frac{kl}{2}\right)$ . The circuit in Figure 2a is then simplified to that shown in Figure 2b.

#### IV. Analysis of Magneto-Electric (ME) Coefficients

The ME coefficient is a measure of a voltage induced by an applied  $\vec{H}$ . There are two methods to express the ME coefficient. These are an induced voltage per unit  $\vec{H}$   $\left(\text{or } \frac{\delta V}{\delta H}\right)$ , or an induced electric field per unit  $\vec{H}$   $\left(\text{or } \frac{\delta E}{\delta H}\right)$ . A number of previous investigations have reported the magneto-electric effect in term of the ME field coefficient or  $\frac{\delta E}{\delta H}$ ; however, the figure of merit of a magnetic field sensor will be related to  $\frac{\delta V}{\delta H}$ . To avoid confusion, we will consistently use these two definitions given above as the representations of ME coefficients.

A high ME coefficient means that the laminate has a high voltage sensitivity to a small  $\vec{H}$ . In evaluations of  $\frac{\delta V}{\delta H}$  to be presented in the following sections, various materials parameters for both terfenol-D and PZT will be required. Table I presents a summary of some of the parameters needed in the analysis.

##### A. Magnetic-electric voltage coefficients

We can derive the ME voltage coefficients from the equivalent circuit in Figure 2b. Under open-circuit conditions, the current  $I_p$  from the piezoelectric layer is zero. Thus, the capacitive load  $C_0$  can be moved to the main circuit loop. Applying Ohm's law to the circuit, the ratio of the output voltage  $V_0$  to the 'magnetically induced voltage'  $\varphi_m H_3$  is

$$\left| \frac{V_0}{\varphi_m H_3} \right| = \left| \frac{\frac{\varphi_p^2}{j\omega C_0}}{Z + \frac{\varphi_p^2}{j\omega C_0}} \right|. \quad (12a)$$

For  $\omega \ll 2\pi f_r = 2\pi \overline{v}/2l$  (where  $f_r$  is the resonance frequency of the laminate), the value of  $\tan(kl/2)$  in the characteristic impedance  $Z$  is  $\sim kl/2$ . As a result, the ME voltage coefficient can be approximated as

$$\left| \frac{dV}{dH_3} \right| = \frac{\varphi_p^2 \varphi_m l}{C_0 A \overline{\rho v}^2 + l \varphi_p^2}; \quad (12b)$$

where the expressions for  $\varphi_m$  and  $\varphi_p$  are given in equation (11). [Note, under small magnetic field excitation,  $|V_0/H_3|$  in equation (12a) can be replaced by  $|dV/dH_3|$ ]. The ME voltage coefficient is dependent upon the magneto-elastic coupling factor  $\varphi_m$  of the terfenol-D layers, and on the square of the electromechanical coupling factor  $\varphi_p$  of the PZT layer. Clearly, high magnetoelastic coupling and high electromechanical coupling are important contributing factors to the design of a high ME voltage coefficient. The ME voltage coefficients in both the L-T and T-T modes have the same general form as given in equation (12); however, the values of  $\varphi_m$  and  $\overline{v}$  are different for the L-T and T-T modes.

### (1) Magneto-electric voltage coefficient in the L-T mode

Under a small ac magnetic field  $H_{ac}$  applied along the longitudinal axis of the laminate, the L-T mode ME voltage coefficient is

$$\left. \frac{dV}{dH_3} \right|_{(L-T)} = \frac{n(1-n)A_{lam}d_{33,m}d_{31,p}^2}{\epsilon_{33}^T s_{11}^E [ns_{11}^E(1-k_{31,p}^2) + (1-n)s_{33}^H]} \quad (13)$$

This is the first quantitative formula for the ME voltage (or field) coefficient. The value of  $\left. \frac{dV}{dH_3} \right|_{(L-T)}$  is proportional to the square of the transverse piezoelectric constants  $d_{31,p}$  and to the longitudinal piezomagnetic constant  $d_{33,m}$ .

Larger cross-sectional areas  $A_{lam}$  result in higher values of  $\left. \frac{dV}{dH_3} \right|_{(L-T)}$ . This has importance to the design of ME

laminates.

In addition, the ME voltage coefficient is strongly related to thickness ratio of the terfenol-D layers,  $n$ . When  $n = 0$  (without piezomagnetism layer), there will not be a magneto-elastic coupling, thus the ME voltage coefficient is zero; when  $n = 1$  (without piezoelectric layer), there will not be an electro-mechanical coupling, thus, the ME voltage coefficient is still zero.

There must be an optimum value for the geometric factor  $n = t_m/t_{lam}$  ( $n_{optim}$ ), which results in a maximum  $\left. \frac{dV}{dH_3} \right|_{(L-T)}$ . This can

be determined from  $d\left(\left. \frac{dV}{dH_3} \right|_{(L-T)}\right)/dn = 0$  to be

$$n_{optim} = \frac{1}{1 + \sqrt{\alpha}}, \text{ where } \alpha = \frac{\epsilon_{33}^S s_{11}^E}{\epsilon_{33}^T s_{33}^H} = (1 - k_{31,p}^2) \frac{s_{11}^E}{s_{33}^H}. \quad (14)$$

Equation (14) is also important to the design of laminate composites. The optimum thickness ratio of terfenol-D layers is related to electromechanical coupling factor  $k_{31,p}$  and the ratio of elastic compliances  $s_{11}^E$  and  $s_{33}^H$  of the magnetostrictive and piezoelectric layers. Figure 3(a) shows the ME voltage coefficient of the L-T mode as a function of  $n$ , which was determined

using the material parameters given in table I. A maximum value for  $\left. \frac{dV}{dH_3} \right|_{(L-T)}$  of ~54 (mV/Oe) was found for our L-T laminate composite (see Figure 1a) of terfenol-D and PZT at  $n \sim 0.64$ .

Table 1 Parameters for calculations of magneto-electric coefficients

	Density (kg/m <sup>3</sup> )	Elastic constants ( $\times 10^{-12}$ )		Piezoelectric/magnetic Constants		Coupling factor $k_{31}$	$\epsilon_{33}^T / \epsilon_0$
PZT	7600	( $s_{11}^E$ ) 14.8	( $s_{33}^E$ ) 16.7	( $d_{33,p}$ )* 600 ( $\times 10^{-12}$ )	( $d_{31,p}$ )* 310 ( $\times 10^{-12}$ )	0.38	2000
T-D	9230	( $s_{11}^H$ ) 125	( $s_{33}^H$ ) 40	( $d_{33,m}$ )** $1.2 \times 10^{-8}$	( $d_{31,m}$ )** $5.8 \times 10^{-9}$		

\* Piezoelectric constants will change under the stress or load applied by piezomagnetic layer

\*\* Piezomagnetic constants are related to the magnetic bias.

### (2) Magneto-electric voltage coefficient in the T-T mode

The T-T mode ME voltage coefficient can also be derived from equation (12b). Under a small  $H_{ac}$  applied along the thickness of the laminate, the T-T mode ME voltage coefficient is

$$\left| \frac{dV}{dH_3} \right|_{(T-T)} = \frac{n(1-n)A_{lam}d_{31,m}d_{31,p}^2}{\epsilon_{33}^T s_{11}^E [ns_{11}^E(1-k_{31,p}^2) + (1-n)s_{11}^H]} \quad (15)$$

The value of  $\left| \frac{dV}{dH} \right|_{(T-T)}$  is proportional to the square of the transverse piezoelectric constants  $d_{31,p}$  and to the transverse piezomagnetic constant  $d_{31,m}$ . Again, the cross-sectional area  $A_{lam}$  of the laminate has a direct effect on the ME voltage coefficient, and also there is an  $n_{optim}$  given as

$$n_{optim} = \frac{1}{1 + \sqrt{\beta}}, \text{ where } \beta = \frac{\epsilon_{33}^S}{\epsilon_{33}^T} \frac{s_{11}^E}{s_{11}^H} = (1 - k_{31,p}^2) \frac{s_{11}^E}{s_{11}^H}. \quad (16)$$

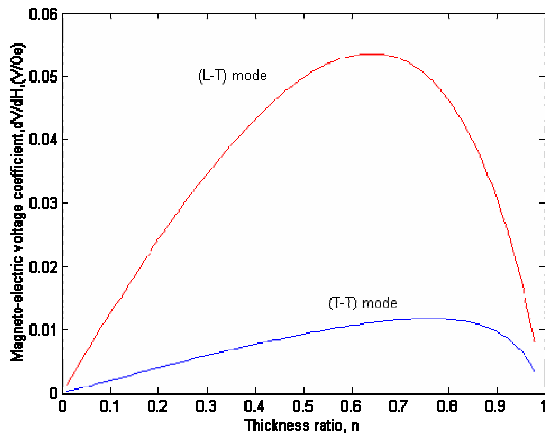
Figure 3(a) also shows the ME voltage coefficient of the T-T mode for our T-T laminate as a function of  $n$ . A maximum value for  $\left| \frac{dV}{dH_3} \right|_{(T-T)}$  of ~11 (mV/Oe) can be seen at  $n \sim 0.76$ . Clearly, the L-T coefficient  $\left| \frac{dV}{dH_3} \right|_{L-T}$  is much larger than the T-T one. The former is ~5x higher than the later. [Note, this will of course depend on magnetic bias.]

## B. Magneto-electric field coefficients

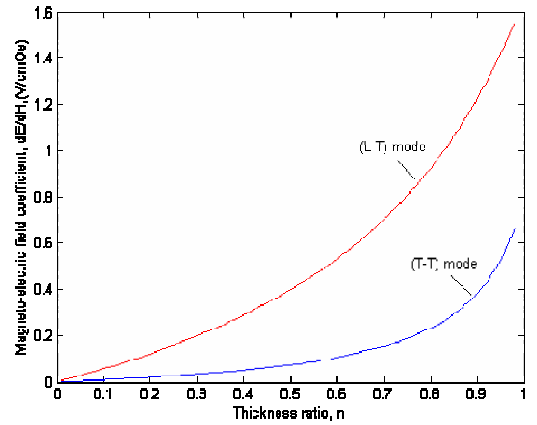
We can calculate the ME field coefficient  $\frac{\delta E}{\delta H}$  by dividing equations (13) and (15) by the thickness of the piezoelectric layer  $t_p$ . Note:  $\frac{A_{lam}}{t_p} = \frac{w_{lam}}{1-n}$ . The L-T  $\left( \left| \frac{dE}{dH} \right|_{L-T} \right)$  and T-T  $\left( \left| \frac{dE}{dH} \right|_{T-T} \right)$  mode ME field coefficients are

$$\left| \frac{dE}{dH_3} \right|_{(L-T)} = \frac{nw_{lam}d_{33,m}d_{31,p}^2}{\epsilon_{33}^T s_{11}^E [ns_{11}^E(1-k_{31,p}^2) + (1-n)s_{33}^H]} \quad (a)$$

$$\left| \frac{dE}{dH_3} \right|_{(T-T)} = \frac{nw_{lam}d_{31,m}d_{31,p}^2}{\epsilon_{33}^T s_{11}^E [ns_{11}^E(1-k_{31,p}^2) + (1-n)s_{11}^H]} \quad (17b)$$



(a) ME voltage coefficients of L-T and T-T laminates calculated using equations (13) and (15).



(b) ME field coefficients of L-T and T-T modes calculated using equations (17a,b).

Figure 3. ME voltage and field coefficients of L-T and T-T laminates.



Figure 3b shows  $\left| \frac{dE}{dH} \right|_{L-T}$  and  $\left| \frac{dE}{dH} \right|_{T-T}$  as a function of the geometric ratio  $n$  for our terfenol-D/ PZT laminates. There is an important difference between the dependence of  $\frac{\delta V}{\delta H}$  and  $\frac{\delta E}{\delta H}$  on  $n$ . Thinner piezoelectric layers (larger  $n$  value) give higher values of  $\frac{\delta E}{\delta H}$ ; however, the voltage output from the laminate is reduced more significantly. This is because  $t_p$  is a function of  $n$ . For magnetic field sensor applications, the important parameter is  $\frac{\delta V}{\delta H}$ , because sensing is done by measurement of induced voltage changes.

## V. Experimental Verification and Discussion

We designed and fabricated terfenol-D/PZT magneto-electric laminates that were long and square. We used the design in Figure 1a for the L-T mode, and that in Figure 1b for the T-T. The dimensions of the terfenol-D layers were  $12.0 \times 6.0 \times 1.0 \text{ mm}^3$ , and that of the PZT layer (PZT-5) was  $12.0 \times 6.0 \times 0.5 \text{ mm}^3$ .

The induced voltage across the PZT plate was then measured as a function of  $H_{ac}$  (a.c magnetic exciting signal) at a measurement frequency of  $10^3 \text{ Hz}$ , using a lock-in amplifier method. The induced ME voltage was a linear function of  $H_{ac}$  for both the L-T and T-T modes. Under a dc magnetic bias of 500 Oe and a  $H_{ac}=1 \text{ Oe}$  applied along the length direction of the laminate, the induced ME voltage of the L-T laminate was 58 mV<sub>p</sub>. Correspondingly, under a  $H_{ac}=1 \text{ Oe}$  applied along the thickness direction of the laminate, the induced ME voltage of the T-T laminate was 9 mV<sub>p</sub>. The corresponding values of

$\left| \frac{dV}{dH} \right|_{L-T}$  and  $\left| \frac{dV}{dH} \right|_{T-T}$  are 58 mV<sub>p</sub>/Oe and 9 mV<sub>p</sub>/Oe, respectively. These measured values coincide well with our analytical predictions, which were based on linear piezoelectric and magnetostrictive constitutive equations, and small magnetic field excitations. In addition, the measurements confirm the prediction that the L-T mode ME voltage coefficient is  $\sim 5\times$  that of the T-T one.

Clearly, the L-T mode has higher ME coefficients than that the T-T one. In particular, under low magnetic bias, the L-T mode had much higher ME effects. More detailed experimental results will be reported separately.

## VI. Conclusion

The magneto-electric equivalent circuit and corresponding magneto-electric coefficients have been derived for terfenol-D/ PZT laminates. Analysis of piezoelectric and piezomagnetic constitutive equations substituted in an equation of motion predict: (i) a significantly higher value of the L-T mode  $\frac{\delta V}{\delta H}$  relative to the T-T ( $\sim 5\times$ ), (ii) a thickness ratio of piezoelectric and magnetostrictive plates that maximizes  $\frac{\delta V}{\delta H}$ , and (iii) that  $dV/dH$  is directly related to the piezoelectric and piezomagnetic constants. Experimental investigations have confirmed predicted values of the magneto-electric voltage coefficients for both L-T and T-T laminates.

## References (omitted)